

KnotTheory

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<< KnotTheory`
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Utilities

```
NormalizePD[K_PD] := K /. i_Integer => Mod[i, 2 Crossings[K]];
FlipCrossing[X[i_, j_, k_, L_]] :=
  If[PositiveQ[X[i, j, k, L]], X[L, i, j, k], X[j, k, L, i]];
FirstEdge[i_, j_] := If[Abs[j - i] == 1, Min[i, j], Max[i, j]];
OrderKnot[K_PD] := Association @@ Flatten @
  (List @@ K /. X[i_, j_, k_, L_] => {i -> X[i, j, k, L], FirstEdge[j, L] -> X[i, j, k, L]});
```

MainProgram

```
HomflyptCoefficient[K_PD, n_] := Module[{m, L, OK, UnderCrossingQ, P, Linking},
  m = 2 Crossings[K]; L = NormalizePD[K]; OK = OrderKnot[L];
  UnderCrossingQ[i_] := (First @ OK[i] == i);
  P[k_, i_, j_] := P[k, i, j] = Expand @ Module[{t},
    Piecewise[{
      {If[k == 0, 1, 0], j - i <= 4},
      {P[k, i + 1, j], t = FirstEdge @@ OK[i][[2, 4]]};
      Not @ UnderCrossingQ[i] || Not[i < t <= j]}],
    If[PositiveQ[OK[i]], -a^2 P[k, i + 1, j] + a^-1 P[k, i, j, t],
      -a^2 P[k, i + 1, j] + a P[k, i, j, t]
    ]];
  Linking[i_, j_, t_] :=
    Sum[If[UnderCrossingQ[1] && (i + 1 <= FirstEdge @@ OK[1][[2, 4]] <= t - 1),
      If[PositiveQ[OK[1]], 1, -1], 0], {1, t + 1, j - 1}];
  P[0, i_, j_, t_] := (-a^-2)^(Linking[i, j, t]) (a + a^-1) P[0, i + 1, t] P[0, t + 1, j];
  P[k_, i_, j_, t_] := Module[{FlipCheck, e, Smooth, D, T, D0},
    FlipCheck[X[a_, b_, c_, d_]] := (e = FirstEdge[b, d];
      (a < i && Not[a < e < i]) || (a >= j && a > e > i));
    Smooth[D_, s_] := Module[{r, d1, d2, d3, A},
      r = FirstEdge @@ OK[s][[2, 4]];
      d1 = i + s - (t + 1); d2 = d1 + t - (r + 1); d3 = d2 + r - (i + 1);
      A = AssociationThread[Range[0, i] ~ Join ~ Range[t + 1, s] ~ Join ~ Range[r + 1, t] ~ Join ~
        Range[i + 1, r] ~ Join ~ Range[s + 1, m - 1] -> Range[0, i] ~ Join ~ Range[i, d1] ~
        Join ~ Range[d1, d2] ~ Join ~ Range[d2, d3] ~ Join ~ Range[d3, m - 5]];
      D /. X[a_, b_, c_, d_] => If[MatchQ[X[a, b, c, d], OK[s]],
        ## &[], X[A[a], A[b], A[c], A[d]]];
      T[j] := (a + a^-1) Sum[P[2 r, i + 1, t] P[k - 2 r, t + 1, j], {r, 0, k/2}];
      T[s_] := If[UnderCrossingQ[s] && i + 1 <= FirstEdge @@ OK[s][[2, 4]] <= t - 1,
        D0 = Smooth[D, s]; D = D /. OK[s] => FlipCrossing[OK[s]];
        If[PositiveQ[OK[s]], a^-1 HomflyptCoefficient[D0, k - 2] - a^-2 T[s + 1],
          a HomflyptCoefficient[D0, k - 2] - a^2 T[s + 1], T[s + 1]];
        D = L /. x_X => If[MatchQ[x, OK[i]], ## &[], If[FlipCheck[x], FlipCrossing[x], x]];
        T[t + 1]
      ];
    P[n, 0, m]
  ];
```

MainProgram

```
HomflyptCoefficient[K_, n_] := HomflyptCoefficient[PD @ K, n];
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